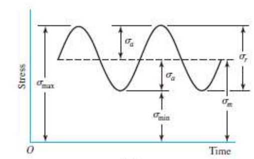
IPE 303
Product Design I

Chapter 6: Fatigue Failure Resulting from Variable Loading



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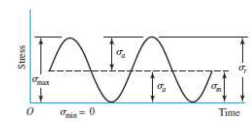

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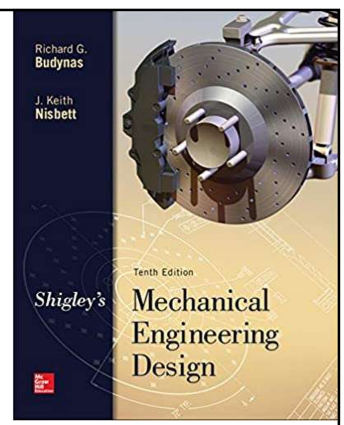

Reference

Shigley's Mechanical Engineering Design, 10th Edition

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6-1 Introduction

- Often, machine members are found to have failed well below the ultimate strength, even below the yield strength. **How?**
- Stresses have been **repeated** a very large number of times, which is known as **Fatigue Failure**.
- Stresses vary with time, or they fluctuate between different levels, known as - variable, repeated, alternating, or fluctuating stresses.
- Most machine elements are subjected to these kind of stresses due to their movement i.e., shafts, gears, bearings, cams & followers, etc.
- Unlike static loading fatigue failures can not be detected before it happens, it is usually sudden and therefore dangerous.

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6-1 Introduction

- Fatigue failure is due to crack formation and propagation. Fatigue cracks usually initiate at locations with high stresses such as discontinuities (hole, notch, scratch, sharp corner, crack, inclusions, etc.).
- Fatigue cracks can also initiate at surfaces having rough surface finish. Thus, all parts subjected to fatigue loading are heat treated and polished in order to increase the fatigue life.

Figure 6–3

Fatigue fracture of an AISI 4320 drive shaft. The fatigue failure initiated at the end of the keyway at points *B* and progressed to final rupture at *C*. The final rupture zone is small, indicating that loads were low. *(From ASM Handbook, Vol. 12:* Fractography, *2nd printing, 1992, ASM International, Materials Park, OH 44073-0002, fig 51, p. 120. Reprinted by permission of ASM International®, www.asminternational.org.)*

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6-3 Fatigue Life Methods

- Fatigue life methods are aimed to determine the life (number of loading cycles) of an element until failure.

- The fatigue life is usually classified according to the number of loading cycles into:
  - **Low cycle fatigue** ($1 \leq N \leq 1000$) and for this low number of cycles, designers sometimes ignore fatigue effects and just use static failure analysis.
  - **High cycle fatigue** ($N > 10^3$)
    - **Finite life**: from $10^3 \rightarrow 10^6$ cycles
    - **Infinite life**: more than 10^6 cycles

- There are **three major fatigue life methods** where each is more accurate for some types of loading or for some materials.
  - **The stress-life method,**
  - The strain-life method,
  - The linear-elastic fracture mechanics method.

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6-3 Fatigue Life Methods

Stress-life method	Strain-life method	Linear-elastic fracture mechanics method
➢ Based on **stress levels** only ➢ **Least accurate** approach, especially for **low-cycle** applications. ➢ However, it is the **most traditional method**, since it is the easiest to implement for a wide range of design applications, has ample supporting data, and represents high-cycle applications adequately.	➢ Involves more detailed analysis of the plastic deformation at localized regions ➢ The stresses and strains are considered for life estimates. ➢ This method is especially good for low-cycle fatigue applications. ➢ In applying this method, several idealizations must be compounded, and so some uncertainties will exist in the results.	➢ Assumes a crack is already present and detected. ➢ Predicts crack growth with respect to stress intensity. ➢ Most practical when applied to large structures in conjunction with computer codes and a periodic inspection program.

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6-4 The Stress-Life Method

Figure 6–9

Test-specimen geometry for the R. R. Moore rotating-beam machine. The bending moment is uniform, $M = Fa$, over the curved length and at the highest-stressed section at the mid-point of the beam.

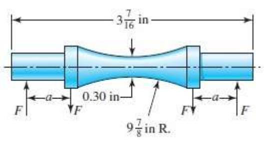
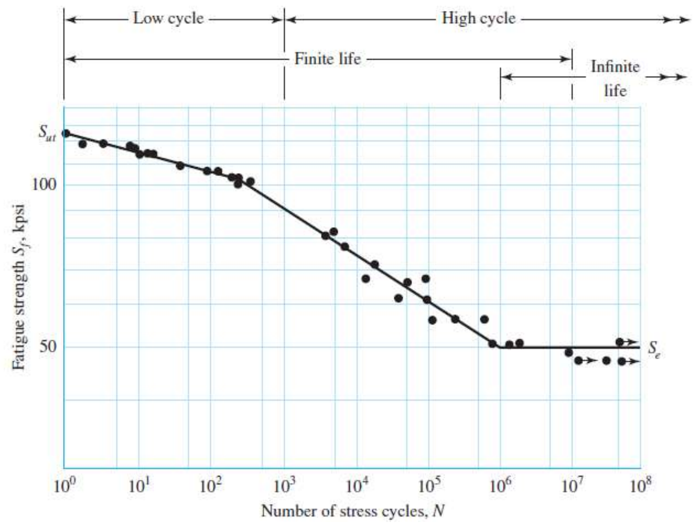


Figure 6–10

An *S-N* diagram plotted from the results of completely reversed axial fatigue tests. Material: UNS G41300 steel, normalized; S_{ut} = 116 kpsi; maximum S_{ut} = 125 kpsi. *(Data from NACA Tech. Note 3866, December 1966.)*



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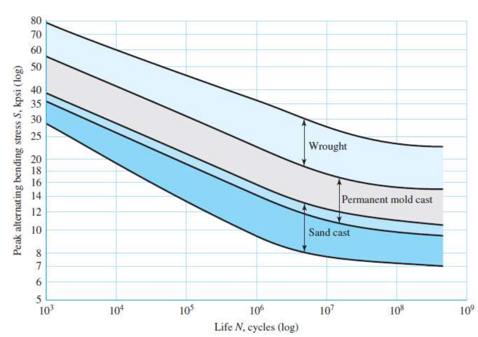
6-4 The Stress-Life Method

- Steel and Titanium alloys have a clear endurance limit;
- The endurance limits for various classes of cast irons, polished or machined, are given in Table A–24.
- However, Aluminum alloys do not have an endurance limit and for such materials the fatigue strength is reported at $5(10^8)$ cycles. (Figure 6-11).

Figure 6–11

S-N bands for representative aluminum alloys, excluding wrought alloys with S_{ut} < 38 kpsi. *(From R. C. Juvinall,* Engineering Considerations of Stress, Strain and Strength. *Copyright © 1967 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*



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6-8 Fatigue Strength

- Approximation of the S-N diagram in the high-cycle region can be found by the following equations which is known as S-N Equations.

$$S_f = a N^b \quad (6\text{–}13)$$

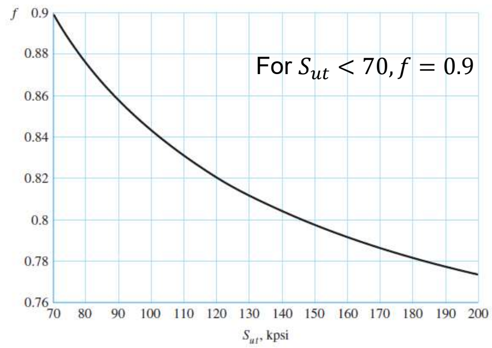
$$a = \frac{(f S_{ut})^2}{S_e} \quad (6\text{–}14)$$

$$b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right) \quad (6\text{–}15)$$

10 base logarithm

Figure 6–18

Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S'_e = 0.5 S_{ut}$ at 10^6 cycles.



If a *completely reversed* stress σ_{rev} is given, setting $S_f = \sigma_{rev}$ in Eq. (6–13), the number of cycles-to-failure can be expressed as

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b} \quad (6\text{–}16)$$

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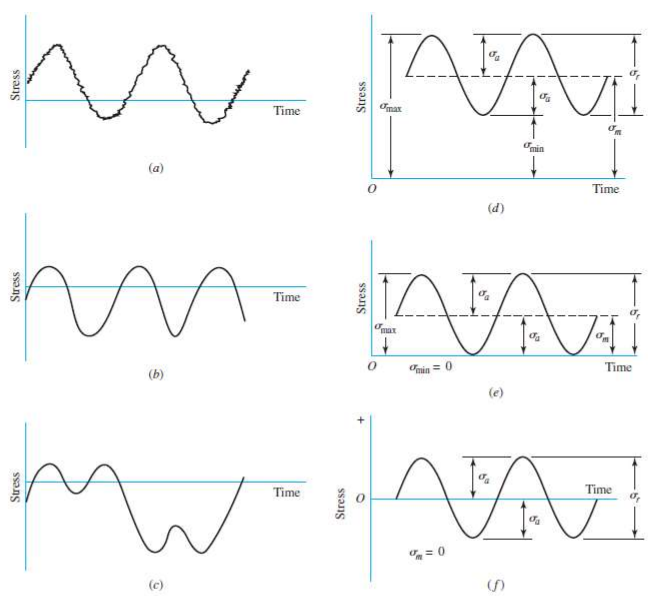
6-8 Fatigue Strength

Note that, the typical S-N diagram, and S-N equations are only applicable for **completely reversed loading!**

Figure 6–23

Some stress-time relations: (*a*) fluctuating stress with high-frequency ripple; (*b* and *c*) nonsinusoidal fluctuating stress; (*d*) sinusoidal fluctuating stress; (*e*) repeated stress; (*f*) completely reversed sinusoidal stress.



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6-8 Fatigue Strength

Table A20

EXAMPLE 6–2 Given a 1050 HR steel, *estimate*

(*a*) the rotating-beam endurance limit at 10^6 cycles.

(*b*) the endurance strength of a polished rotating-beam specimen corresponding to 10^4 cycles to failure

(*c*) the expected life of a polished rotating-beam specimen under a completely reversed stress of 55 kpsi.

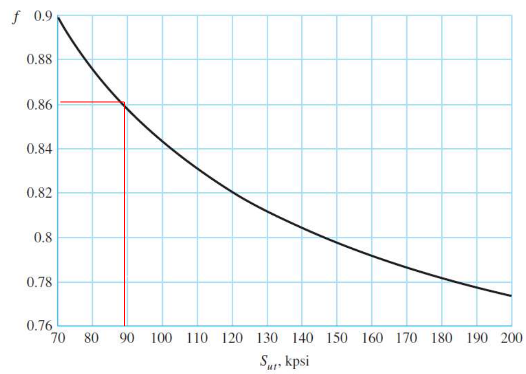
$$S_f = a\,N^b \qquad (6\text{–}13)$$

$$a = \frac{(f\,S_{ut})^2}{S_e} \qquad (6\text{–}14)$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) \qquad (6\text{–}15)$$

Figure 6–18

Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S'_e = 0.5S_{ut}$ at 10^6 cycles.



If a *completely reversed* stress σ_{rev} is given, setting $S_f = \sigma_{rev}$ in Eq. (6–13), the number of cycles-to-failure can be expressed as

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b} \qquad (6\text{–}16)$$

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6-7 The Endurance Limit

- The common practice when designing elements subjected to high fatigue is to make sure that the fatigue stress level in the element is below the endurance limit of the material being used.
- The endurance limit can be related to some mechanical properties which are easier to find, i.e.,

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > \text{kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \qquad (6\text{–}8)$$

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \qquad (6\text{–}18)$$

k_a = surface condition modification factor

k_b = size modification factor

k_c = load modification factor

k_d = temperature modification factor

k_e = reliability factor[13]

k_f = miscellaneous-effects modification factor

S'_e = rotary-beam test specimen endurance limit

S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

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6-9 Endurance Limit Modifying Factors

Surface Condition Factor (k_a)

- The rotating-beam test specimens are **highly polished**. A rough surface finish will reduce the endurance limit because there will be a higher potential for crack initiation.
- The surface condition modification factor depends on the surface finish of the part (ground, machined, as forged, etc.) and on the tensile strength of the material. It is given as:

$$k_a = aS_{ut}^b \qquad (6\text{–}19)$$

where S_{ut} is the minimum tensile strength and a and b are to be found in Table 6–2.

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

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6-9 Endurance Limit Modifying Factors

Size Factor (k_b)

- The rotating-beam specimens have a specific (small) diameter (7.6mm). Parts of larger size are more likely to contain flaws and to have more non-homogeneities.

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \qquad (6\text{–}20)$$

For axial loading there is no size effect, so

$$k_b = 1 \qquad (6\text{–}21)$$

- When a member with **circular cross-section is not rotating**, we use an effective diameter value instead of the actual diameter.

$$d_e = 0.37d$$

- For **other nonrotating cross-sections**, $A_{0.95\sigma}$ is found using **Table 6-3,** and d_e can be found using –

$$A_{0.95} = 0.0766d^2$$

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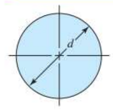
6-9 Endurance Limit Modifying Factors

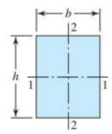
Size Factor (k_b)

Table 6–3

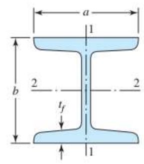
$A_{0.95\sigma}$ Areas of Common Nonrotating Structural Shapes



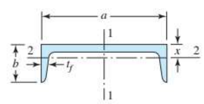
$$A_{0.95\sigma} = 0.01046d^2$$
$$d_e = 0.370d$$



$$A_{0.95\sigma} = 0.05hb$$
$$d_e = 0.808\sqrt{hb}$$



$$A_{0.95\sigma} = \begin{cases} 0.10at_f & & \text{axis 1-1} \\ 0.05ba & t_f > 0.025a & \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b - x) & \text{axis 2-2} \end{cases}$$

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6-9 Endurance Limit Modifying Factors

Loading Factor (k_c)

- The rotating-beam specimen is loaded in bending. Other types of loading will have a different effect.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{–}26)$$

$k_c = 1$ when torsion is combined with other loading type.

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6-9 Endurance Limit Modifying Factors

Temperature Factor (k_d)

- When the operating temperature is below room temperature, the material becomes more brittle. When the temperature is high the yield strength decreases, and the material becomes more ductile.

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \quad (6\text{–}27)$$

where $70 \le T_F \le 1000°F$.

$$k_d = 0.9877 + 0.6507(10^{-3})T_c - 0.3414(10^{-5})T_c^2 + 0.5621(10^{-8})T_c^3 - 0.6246(10^{-11})T_c^4$$

For $40 \le T_c \le 540\,°C$

Table 6–4

Effect of Operating Temperature on the Tensile Strength of Steel.* (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature; $0.099 \le \hat{\sigma} \le 0.110$)

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

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6-9 Endurance Limit Modifying Factors

Reliability Factor (k_e)

Table 6–5

Reliability Factors k_e Corresponding to 8 Percent Standard Deviation of the Endurance Limit

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

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6-9 Endurance Limit Modifying Factors

Miscellaneous-Effects Factor (k_f)

- It is used to account for the reduction of endurance limit due to **all other effects (such as residual stress, corrosion, cyclic frequency, metal spraying, etc.).**
- However, those effects are not fully characterized and usually not accounted for. Thus, we use $k_f = 1$.

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6-9 Endurance Limit Modifying Factors

EXAMPLE 6–3 A steel has a minimum ultimate strength of 520 MPa and a machined surface. Estimate k_a.

EXAMPLE 6–4 A steel shaft loaded in bending is 32 mm in diameter, abutting a filleted shoulder 38 mm in diameter. The shaft material has a mean ultimate tensile strength of 690 MPa. Estimate the Marin size factor k_b if the shaft is used in
(*a*) A rotating mode.
(*b*) A nonrotating mode.

EXAMPLE 6–5 A 1035 steel has a tensile strength of 70 kpsi and is to be used for a part that sees 450°F in service. Estimate the Marin temperature modification factor and $(S_e)_{450°}$ if
(*a*) The room-temperature endurance limit by test is $(S'_e)_{70°} = 39.0$ kpsi.
(*b*) Only the tensile strength at room temperature is known.

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6-9 Endurance Limit Modifying Factors

EXAMPLE 6–8 A 1015 hot-rolled steel bar has been machined to a diameter of 1 in. It is to be placed in reversed axial loading for 70 000 cycles to failure in an operating environment of 550°F. Using ASTM minimum properties, and a reliability of 99 percent, estimate the endurance limit and fatigue strength at 70 000 cycles.

Table A–20

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ($\frac{3}{4}$ to $1\frac{1}{4}$ in). These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.] *Source:* 1986 SAE Handbook, p. 2.15.

1 UNS No.	2 SAE and/or AISI No.	3 Process-ing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111

Table 6–2

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Surface Finish	Factor a S_{ut}, kpsi	 S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
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150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
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450	0.843	900	0.797
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Table 6–5

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6-9 Endurance Limit Modifying Factors

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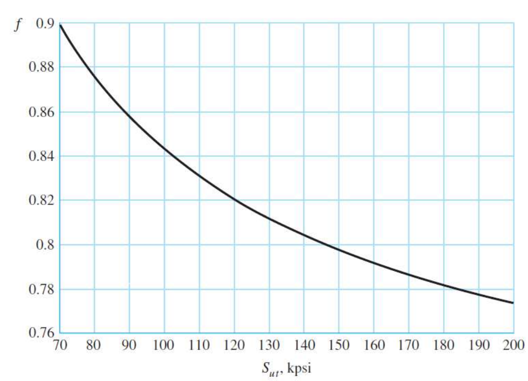
$$S_f = a\,N^b$$

$$a = \frac{(f\,S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right)$$

Figure 6–18

Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S'_e = 0.5S_{ut}$ at 10^6 cycles.



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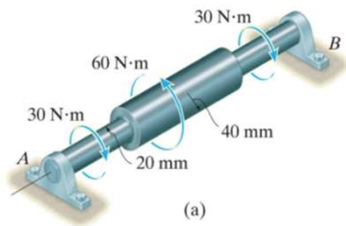
6-10 Stress Concentration and Notch Sensitivity



(a)

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6-10 Stress Concentration and Notch Sensitivity

- Any **discontinuity** in a machine part alters the stress distribution in the neighborhood of the discontinuity.
- For example, Rotating shafts must have **shoulders** designed on them so that the bearings can be properly seated, shafts must have **key slots** machined into them for securing pulleys and gears etc.
- Such discontinuities are called **stress raisers**, and the regions in which they occur are called areas of **stress concentration**.

$$K_t = \frac{\sigma_{max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{max}}{\tau_0}$$

Table A-15
Charts of Theoretical Stress-Concentration Factors K_t^*

- The nominal stress σ_0 *or* τ_0 is the stress calculated by using the elementary stress equations and the net area, or net cross section.
- For materials under fatigue loading, the maximum stress near a notch (hole, fillet, etc.) is –

$$\sigma_{max} = K_f \sigma_0 \quad \text{or} \quad \tau_{max} = K_{fs} \tau_0 \qquad (6\text{–}30)$$

- K_f is the **fatigue stress concentration factor** which is a reduced value of the **theoretical stress concentration factor** (K_t) because of the difference in material sensitivity to the presence of notches.

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6-10 Stress Concentration and Notch Sensitivity

$$K_f = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch-free specimen}} \qquad (a)$$

- Notch sensitivity (q) can be used in the relation between K_f *and* K_t as follows.

$$K_f = 1 + q(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1) \qquad (6\text{–}32)$$

The value of q ranges from *0* to *1*

$q = 0$ ➔ $K_f = 1$ (*material is not sensitive*)

$q = 1$ ➔ $K_f = K_t$ (*material is fully sensitive*)

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6-10 Stress Concentration and Notch Sensitivity

- For Steels and Aluminum (2024) the notch sensitivity for Bending and Axial loading can be found from Figure 6-20 and for Torsion is found from Figure 6-21.

Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the r = 0.16-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*

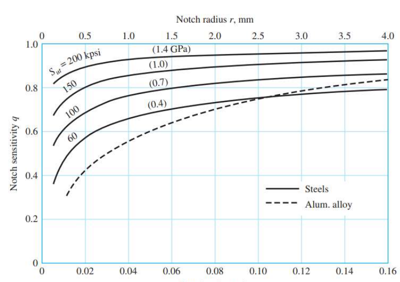
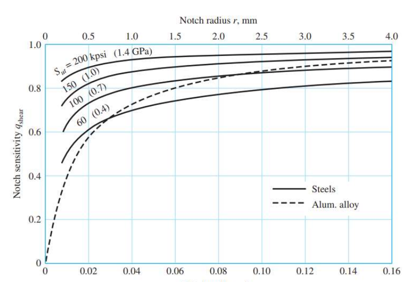


Figure 6–21

Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of q_{shear} corresponding to r = 0.16 in (4 mm).



- Alternatively, using The Neuber equation -

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}}$$

where, r : radius

$\sqrt{a}$: is a material constant known as the Neuber constant.

Bending or axial: $\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35*a*)

Torsion: $\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35*b*)

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6-10 Stress Concentration and Notch Sensitivity

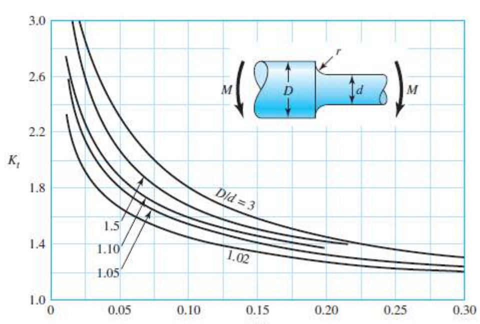
- For cast iron, the notch sensitivity is very low from 0 to 0.2, but to be conservative it is recommended to use $q = 0.2$
- For simple loading, K_f can be multiplied by the stress value.
- However, for combined loading each type of stress has to be multiplied by its corresponding K_f value.

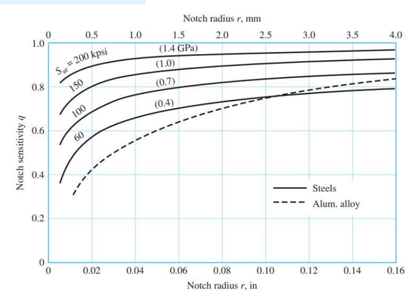
6-10 Stress Concentration and Notch Sensitivity

EXAMPLE 6–6 A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:
(*a*) Figure 6–20.
(*b*) Equations (6–33) and (6–35).

Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.





6-10 Stress Concentration and Notch Sensitivity

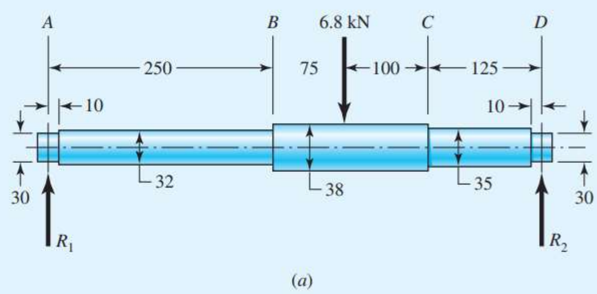
EXAMPLE 6–7 For the step-shaft of Ex. 6–6, it is determined that the fully corrected endurance limit is $S_e = 280$ MPa. Consider the shaft undergoes a fully reversing nominal stress in the fillet of $(\sigma_{rev})_{nom} = 260$ MPa. Estimate the number of cycles to failure.

6-10 Stress Concentration and Notch Sensitivity

EXAMPLE 6–9 Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

Figure 6–22

(*a*) Shaft drawing showing all dimensions in millimeters; all fillets 3-mm radius. The shaft rotates and the load is stationary; material is machined from AISI 1050 cold-drawn steel. (*b*) Bending-moment diagram.



6-11 Characterizing Fluctuating Stresses

- Fluctuating stresses in machinery often take the form of a **sinusoidal patterns**.
- However, the shape of the wave is not something we are interested in, rather the **amplitude and midrange value are very important**.
- The components of stresses are –

$\sigma_{\min}$ = minimum stress
$\sigma_{\max}$ = maximum stress
σ_a = amplitude component
σ_m = midrange component
σ_r = range of stress
σ_s = static or steady stress

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

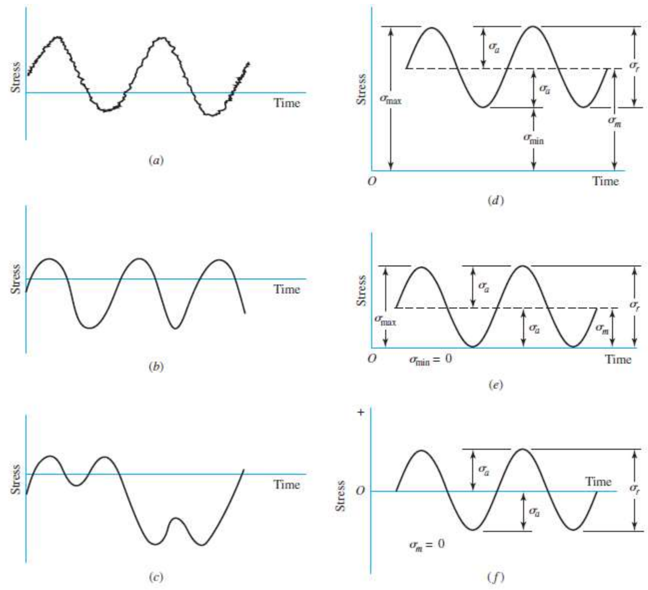
$$\sigma_a = \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right|$$

the *stress ratio* $R = \dfrac{\sigma_{\min}}{\sigma_{\max}}$

the *amplitude ratio* $A = \dfrac{\sigma_a}{\sigma_m}$

Figure 6–23

Some stress-time relations: (*a*) fluctuating stress with high-frequency ripple; (*b* and *c*) nonsinusoidal fluctuating stress; (*d*) sinusoidal fluctuating stress; (*e*) repeated stress; (*f*) completely reversed sinusoidal stress.



6-11 Characterizing Fluctuating Stresses

- The steady, or static, stress is not the same as the midrange stress. In fact, it may have any value between σ_{min} and σ_{max}.
- The steady stress exists because of a fixed load or preload applied to the part, and it is usually independent of the varying portion of the load.
- Note that, In presence of a notch, both nominal values of amplitude and midrange components of stresses (i.e., σ_{a0} and σ_{m0}) must be multiplied by fatigue stress-concentration factor K_f.

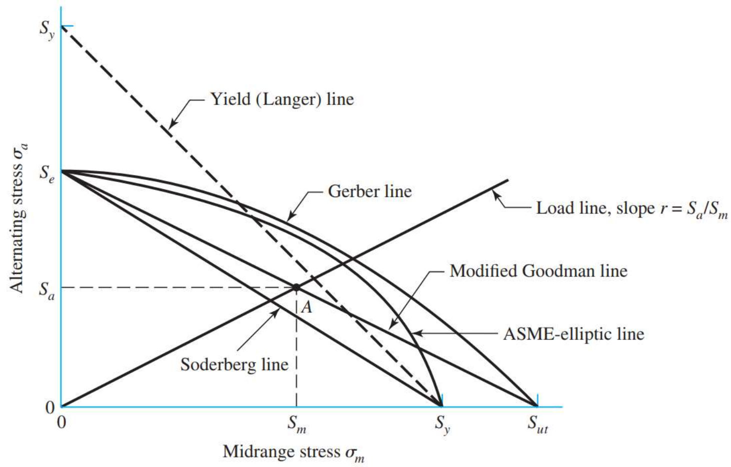
$$\sigma_a = K_f \times \sigma_{ao} \text{ and } \sigma_m = K_f \times \sigma_{mo}$$

6-12 Fatigue Failure Criteria for Fluctuating Stress

- When a machine element is subjected to completely reversed stress ($\sigma_m = 0$) the endurance limit is obtained from the rotating-beam test.
- However, when $\sigma_m \neq 0$, the situation is different, and a **fatigue failure criterion** is needed.

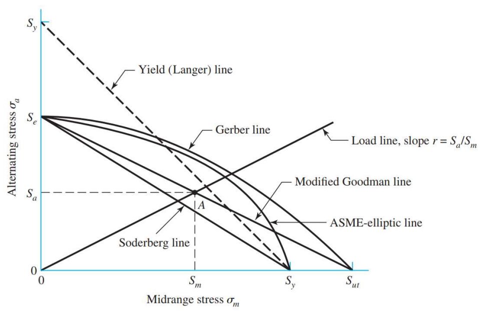
Figure 6–27

Fatigue diagram showing various criteria of failure. For each criterion, points on or "above" the respective line indicate failure. Some point A on the Goodman line, for example, gives the strength S_m as the limiting value of σ_m corresponding to the strength S_a, which, paired with σ_m, is the limiting value of σ_a.



6-12 Fatigue Failure Criteria for Fluctuating Stress

- When $\sigma_m = 0$ & $\sigma_a \neq 0$, this will be a **completely reversed fluctuating stress.**
- When $\sigma_m \neq 0$ & $\sigma_a = 0$, this will be a **static stress.**
- Any combination of σ_m & σ_a will fall between the two extremes (completely reversed & static) and we need to use above criteria.



Soderberg $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n} \quad (6\text{–}45)$$

Equation (6–41), the modified Goodman line, becomes

mod-Goodman $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad (6\text{–}46)$$

Equation (6–42), the Gerber line, becomes

Gerber $$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1 \quad (6\text{–}47)$$

Equation (6–43), the ASME-elliptic line, becomes

ASME-elliptic $$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1 \quad (6\text{–}48)$$

The design equation for the Langer first-cycle-yielding is

Langer static yield $$\sigma_a + \sigma_m = \frac{S_y}{n} \quad (6\text{–}49)$$

6-12 Fatigue Failure Criteria for Fluctuating Stress

The failure criteria are used in conjunction with a load line, $r = S_a/S_m = \sigma_a/\sigma_m$. Principal intersections are tabulated in Tables 6–6 to 6–8. Formal expressions for fatigue factor of safety are given in the lower panel of Tables 6–6 to 6–8. The first row of each table corresponds to the fatigue criterion, the second row is the static Langer criterion, and the third row corresponds to the intersection of the static and

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6-12 Fatigue Failure Criteria for Fluctuating Stress

Table 6–6

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Modified Goodman and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_a = \frac{rS_eS_{ut}}{rS_{ut} + S_e}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{rS_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_m = \frac{(S_y - S_e)S_{ut}}{S_{ut} - S_e}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{crit} = S_a/S_m$

Fatigue factor of safety

$$n_f = \frac{1}{\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}}$$

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6-12 Fatigue Failure Criteria for Fluctuating Stress

Table 6–7

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Gerber and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{rS_{ut}}\right)^2}\right]$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{rS_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \frac{S_{ut}^2}{2S_e}\left[1 - \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2\left(1 - \frac{S_y}{S_e}\right)}\right]$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m$, $r_{crit} = S_a/S_m$

Fatigue factor of safety

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

6-12 Fatigue Failure Criteria for Fluctuating Stress

Table 6–8

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for ASME-Elliptic and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = \sqrt{\frac{r^2 S_e^2 S_y^2}{S_e^2 + r^2 S_y^2}}$
Load line $r = S_a/S_m$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{rS_y}{1 + r}$
Load line $r = S_a/S_m$	$S_m = \frac{S_y}{1 + r}$
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = 0, \frac{2S_y S_e^2}{S_e^2 + S_y^2}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_m = S_y - S_a$, $r_{crit} = S_a/S_m$

Fatigue factor of safety

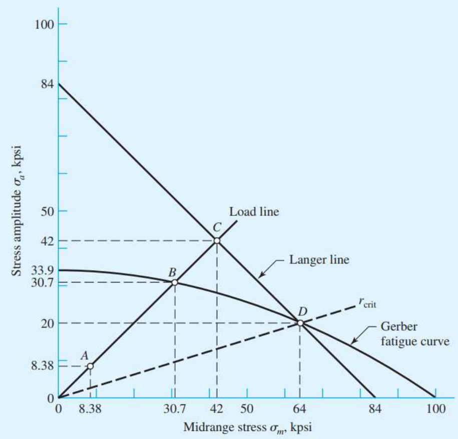
$$n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$$

6-12 Fatigue Failure Criteria for Fluctuating Stress

EXAMPLE 6–10 A 1.5-in-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 16 kip. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line and (*b*) the ASME-elliptic fatigue line.

Figure 6–28

Principal points *A*, *B*, *C*, and *D* on the designer's diagram drawn for Gerber, Langer, and load line.



6-12 Fatigue Failure Criteria for Fluctuating Stress

EXAMPLE 6–12 A steel bar undergoes cyclic loading such that $\sigma_{max} = 60$ kpsi and $\sigma_{min} = -20$ kpsi. For the material, $S_{ut} = 80$ kpsi, $S_y = 65$ kpsi, a fully corrected endurance limit of $S_e = 40$ kpsi, and $f = 0.9$. Estimate the number of cycles to a fatigue failure using:
(*a*) Modified Goodman criterion.
(*b*) Gerber criterion.

6-13 Torsional Fatigue Loading

- For shafts that are subjected to fluctuating **shear stress** with non-zero mean, a fatigue criterion (ASME elliptic, Gerber, etc.) needs to be used.
- It should be noted that the endurance limit S_e already accounts for the torsional loading since $K_c = 0.59$ is used in such case.
- Similarly, the yield or ultimate strengths for shear need to be corrected since the "shear yield strength" (S_{sy}) or the "shear ultimate strength" (S_{us}) need to be used and those are found as:

$$S_{sy} = 0.577 S_y \; and \; S_{us} = 0.67 S_{ut}$$

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6-14 Combination of Loading Modes

- Involves cases where there are combinations of different types of loading, such as combined bending, torsion, and axial.
- The distortion energy failure theory proved to be a satisfactory method of combining the multiple stresses into a single equivalent von Mises stress, $\sigma' = \sqrt{\sigma^2 + 3\tau^2}$

$$\sigma'_a = \left\{ \left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}}\frac{(\sigma_a)_{\text{axial}}}{0.85} \right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}}\right]^2 \right\}^{1/2} \quad (6\text{–}55)$$

$$\sigma'_m = \left\{ \left[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}} \right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}}\right]^2 \right\}^{1/2} \quad (6\text{–}56)$$

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6-13 Torsional Fatigue Loading

EXAMPLE 6–14 A shaft is made of 42- × 4-mm AISI 1018 cold-drawn steel tubing and has a 6-mm-diameter hole drilled transversely through it. Estimate the factor of safety guarding against fatigue and static failures using the Gerber and Langer failure criteria for the following loading conditions:
(*a*) The shaft is rotating and is subjected to a completely reversed torque of 120 N · m in phase with a completely reversed bending moment of 150 N · m.
(*b*) The shaft is subjected to a pulsating torque fluctuating from 20 to 160 N · m and a steady bending moment of 150 N · m.

Table 6–7
Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Gerber and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{rS_{ut}}\right)^2}\right]$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{rS_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \frac{S_{ut}^2}{2S_e}\left[1 - \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2\left(1 - \frac{S_y}{S_e}\right)}\right]$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m$, $r_{crit} = S_a/S_m$
Fatigue factor of safety	
$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$	$\sigma_m > 0$

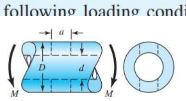
$$\sigma'_a = \left\{\left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}}\frac{(\sigma_a)_{\text{axial}}}{0.85}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}}\right]^2\right\}^{1/2} \quad (6\text{–}55)$$

$$\sigma'_m = \left\{\left[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}}\right]^2\right\}^{1/2} \quad (6\text{–}56)$$

6-13 Torsional Fatigue Loading

EXAMPLE 6–14 A shaft is made of 42- × 4-mm AISI 1018 cold-drawn steel tubing and has a 6-mm-diameter hole drilled transversely through it. Estimate the factor of safety guarding against fatigue and static failures using the Gerber and Langer failure criteria for the following loading conditions:

Table A–16
Approximate Stress-Concentration Factor K_t of a Round Bar or Tube with a Transverse Round Hole and Loaded in Bending
Source: R. E. Peterson, *Stress-Concentration Factors*, Wiley, New York, 1974, pp. 146, 235.



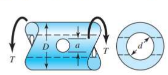
The nominal bending stress is $\sigma_0 = M/Z_{net}$ where Z_{net} is a reduced value of the section modulus and is defined by

$$Z_{net} = \frac{\pi A}{32D}(D^4 - d^4)$$

Values of A are listed in the table. Use $d = 0$ for a solid bar

	d/D					
	0.9		0.6		0	
a/D	A	K_t	A	K_t	A	K_t
0.050	0.92	2.63	0.91	2.55	0.88	2.42
0.075	0.89	2.55	0.88	2.43	0.86	2.35
0.10	0.86	2.49	0.85	2.36	0.83	2.27
0.125	0.82	2.41	0.82	2.32	0.80	2.20
0.15	0.79	2.39	0.79	2.29	0.76	2.15
0.175	0.76	2.38	0.75	2.26	0.72	2.10
0.20	0.73	2.39	0.72	2.23	0.68	2.07
0.225	0.69	2.40	0.68	2.21	0.65	2.04
0.25	0.67	2.42	0.64	2.18	0.61	2.00
0.275	0.66	2.48	0.61	2.16	0.58	1.97
0.30	0.64	2.52	0.58	2.14	0.54	1.94

Table A–16 (*Continued*)
Approximate Stress-Concentration Factors K_{ts} for a Round Bar or Tube Having a Transverse Round Hole and Loaded in Torsion *Source:* R. E. Peterson, *Stress-Concentration Factors*, Wiley, New York, 1974, pp. 148, 244.



The maximum stress occurs on the inside of the hole, slightly below the shaft surface. The nominal shear stress is $\tau_0 = TD/2J_{net}$, where J_{net} is a reduced value of the second polar moment of area and is defined by

$$J_{net} = \frac{\pi A(D^4 - d^4)}{32}$$

Values of A are listed in the table. Use $d = 0$ for a solid bar.

	d/D									
	0.9		0.8		0.6		0.4		0	
a/D	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}
0.05	0.96	1.78							0.95	1.77
0.075	0.95	1.82							0.93	1.71
0.10	0.94	1.76	0.93	1.74	0.92	1.72	0.92	1.70	0.92	1.68
0.125	0.91	1.76	0.91	1.74	0.90	1.70	0.90	1.67	0.89	1.64
0.15	0.90	1.77	0.89	1.75	0.87	1.69	0.87	1.65	0.87	1.62
0.175	0.89	1.81	0.88	1.76	0.87	1.69	0.86	1.64	0.85	1.60
0.20	0.88	1.96	0.86	1.79	0.85	1.70	0.84	1.63	0.83	1.58
0.25	0.87	2.00	0.82	1.86	0.81	1.72	0.80	1.63	0.79	1.54
0.30	0.80	2.18	0.78	1.97	0.77	1.76	0.75	1.63	0.74	1.51
0.35	0.77	2.41	0.75	2.09	0.72	1.81	0.69	1.63	0.68	1.47
0.40	0.72	2.67	0.71	2.25	0.68	1.89	0.64	1.63	0.63	1.44